

Electric Fields in material space

$\epsilon_0$ : permittivity in vacuum (free space)

For electric properties of bulk material

$\sigma$ : conductivity (conductors)

$\epsilon$ : permittivity (dielectrics)

Perfect conductors

$$\sigma \rightarrow \infty$$

Perfect insulators

$$\sigma \rightarrow 0$$

Ohm's Law

$$\vec{J} = \sigma \vec{E}$$

Types of currents

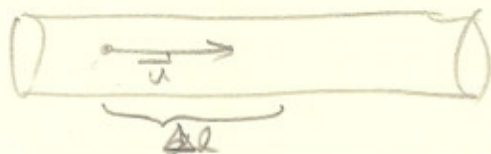
Convection

Conduction

Displacement

Current: A flow of charge through space

$$I = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$$



In an interval  $\Delta t$  ;

$$\Delta Q = \rho_v \underbrace{u \Delta t}_{} S$$

surface  
being  
considered

charge  
density

charge density  
volume of tube

$$I = \frac{\Delta Q}{\Delta t} = \frac{\rho_v u \Delta t S}{\Delta t} = \rho_v u S$$

$\vec{J}$  : current density ( $\rho_v \vec{u}$ )

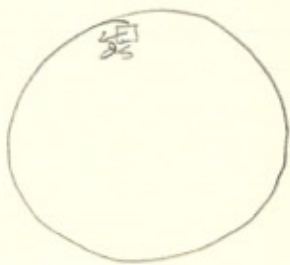
$$\frac{I}{S} = |\vec{J}|$$

Now ...

$$I = \rho_v u S = \rho_v \vec{u} \cdot \vec{S}$$

Definition of current then is:

$$I = \int_S \vec{J} \cdot d\vec{S}$$



$\vec{J} \cdot d\vec{S}$  is a small  
amount of current  
through  $d\vec{S}$

Every charge carrier is subject to a force

$$\vec{F} = q\vec{E} = m\vec{a} \quad (\text{in free space})$$

In conductors for conduction currents Newton's Law does not apply.

Collisions with atoms produce random scatterings

$$q\vec{E} = \frac{m\vec{u}}{\tau} \quad (\text{works better})$$

$\tau$ : average time between collisions.

Aside

Mean free Path  $|\vec{u}|\tau$

$$P = nq$$

depends on 2 factors

- Ⓐ density of carrier
- Ⓑ Change of carrier.

$$\vec{J} = P\vec{u} = nq \left( \frac{q\vec{E}\tau}{m} \right) = \frac{n\tau q^2}{m} \vec{E}$$

$$\vec{J} = \alpha \vec{E}$$

$$\nwarrow \frac{n\tau q^2}{m}$$



In ideal conductors electrons are free

$$\tau \rightarrow \infty \quad \therefore \sigma \rightarrow \infty$$

In dielectrics; electrons are not free

$$\tau = 0 \quad \therefore \sigma \rightarrow 0$$

In a perfect conductor

$$\vec{J} = \infty \quad \& \quad \sigma = \infty$$

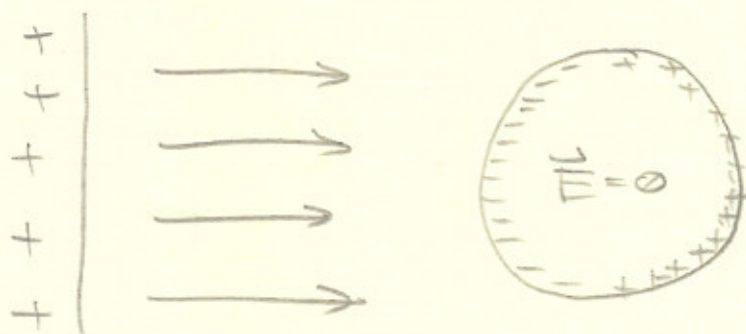
$$\therefore \vec{E} = \infty$$

In a perfect conductor in a static case situation:

$$\vec{E} = 0$$

$$V_{A,B} = - \int_A^B \vec{E} \cdot d\vec{l} = 0$$

Imagine I plunge a perfect conductor into an external field.



$$\oint \vec{D} \cdot d\vec{S} = q_{\text{enclosed}}$$

$$\oint \epsilon_0 \vec{E} \cdot d\vec{S} = q_{\text{enclosed}}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$q_{\text{enclosed}} = 0$  inside conductor

$$V_{AB} = 0 = - \int \vec{E} \cdot d\vec{\ell} \quad (\text{for } A, B \text{ inside conductor})$$

Tutorial

EX

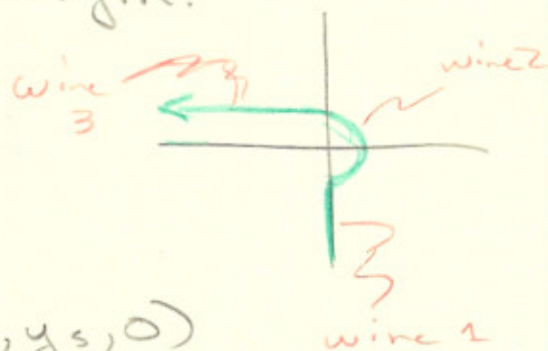
A infinitely long thin wire arriving from  $-\infty$  along  $\hat{y}$  is bent in a semicircle of radius "a" centered at the origin; and returns to negative infinity along the line  $y=a$ . If wires carry a uniform charge density  $\lambda$ , C/m; find electric potential at the origin.

SOL for  $\vec{R}$

$$\vec{R} = \vec{R}_p - \vec{R}_s$$

$$\vec{R} = (0, 0, 0) - (0, y_s, 0)$$

$$\vec{R} = (0, -y_s, 0)$$



Solve

$$\frac{\vec{A}_r}{R^2} = \frac{\vec{R}}{|\vec{R}|^3} = -\frac{1}{y_s} \hat{y}$$

$$\vec{E} = \int_{-\infty}^a \frac{d_0}{4\pi\epsilon_0} \left( -\frac{1}{y_s} \right) dy_s \hat{y}$$

$$= \frac{-d_0}{4\pi\epsilon_0 a} \hat{y}$$

for wire 2

so on sub the totals.